UNIVERSITY OF SOUTH ALABAMA
DEPARTMENT OF MATHEMATICS AND STATISTICS

Comprehensive Examination
Probability
Summer Quarter 1997

I) State the definitions of the following terms:

A) σ-algebra
B) The set of Borel sets
C) random variables
D) probability space

II) State the following theorems

A) The Central Limit Theorem
B) Chebyshev's Inequality
C) The Strong Law of Large Numbers
D) Baye's Formula

III) Suppose that $X \sim N(0, 1)$.

1. Prove that X and X^2 are **not** independent.
2. Prove that $\text{Cov}(X, X^2 = 0$.
3. Give an example of two random variables that are not independent but have covariance 0.
4. Derive the density function for X^2.

IV) Do the following problems.

A) If $A_1, A_2, \cdots$ is an infinite sequence of events such that $P(A_i) = 0, i = 1, 2, \cdots$, prove that $P(\bigcup\limits_{i=1}^{\infty})A_i) = 0$.

B) If $B_1, B_2, \cdots$ is an infinite sequence of events such that $P(B_i) = 1, i = 1, 2, \cdots$, prove that $P(\bigcap\limits_{i=1}^{\infty} B_i) = 1$.

V) If $X \sim \Gamma(s, \gamma)$, (gamma) prove that the moment generating function for X is $(\frac{\gamma}{\gamma - t})^s$.